

FREE-CONVECTIVE HEAT TRANSFER FROM CYLINDRICAL SURFACES WITH TRANSVERSE SECTIONAL FINNING

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A mathematical model has been developed and numerical calculations of heat transfer have been performed for a horizontal heated cylindrical base with transverse sectional finning under the conditions of free-convective flow of an external heat-transfer agent. The effect of concurrent flows from lower fins on the heat transfer of upper fins has been studied. Local thermal characteristics of individual sectional fins on different portions of the cylindrical base and total heat fluxes removed by a finned surface have been determined. Results of the calculation have been compared to experimental data and satisfactory agreement of them has been obtained.

Heat exchangers made of finned tubes have found wide application in designing different heat-exchange equipment. In designing it, transverse annular or helical finning is used to enhance heat transfer. Recent investigations have shown that the efficiency of such equipment can be improved by using discontinuous finning, which is attained by dissecting either individual annular fins or a metal strip used for creation of helical transverse finning. The technology of production of tubes with transverse sectional finning has been mastered well, and surfaces with such finning have passed evaluation tests in designing a number of heat exchangers. It has been shown [1, 2] that surfaces with sectional finning offer some advantages over similar structures with continuous transverse finning. In flow of the external heat-transfer agent past sectional fins, we have the separation of a boundary layer on the initial portions of its formation where the local values of the heat-transfer coefficients are high. Tubes with sectional finning are used in heat exchangers with both forced and free-convective modes of flow of the external heat-transfer agent. At the same time, the conditions of heat transfer of sectional finning on a cylindrical base have not been studied adequately. Methods of thermal calculation of such surfaces are also poorly developed. Below, we give thermal calculation of a cylinder with transverse discrete finning under the conditions of natural convection.

We consider a horizontal cylindrical heated surface with transverse sectional finning (Fig. 1) which is cooled in free-convective flow of the external heat-transfer agent. Individual fins of the surface considered have small cross-sectional dimensions which are much smaller than the fin height ($h \gg l$). Depending on the position of the individual fins on the surface of the cylindrical base, they are arranged at a certain angle γ with the vertical, and the angular distance between neighboring fins is $\Delta\varphi$ (Fig. 2). The boundary layer on the surface of small fins is laminar, and it is formed in different ways depending on the angle of inclination of the fins. In particular, on the lower fin ($\gamma = 180^\circ$) the boundary layer is formed near the end face and increases as the cylinder base is approached. For other fins arranged on the side surfaces of the cylindrical base ($0 < \gamma < 180^\circ$), the boundary layer is formed near the lower edge of the end face and increases along the surface at an angle γ with the coordinate Oz and then separates at the upper edge of the side or upper end face of the fin. For annular or helical sectional finning with a small interfin distance, almost all the fins, except for those lying on the lower portion of the base, are in the concurrent flow of a hot heat-transfer agent which is formed in separation of the boundary layer from the lower-lying fins. We assume that, due to a small vertical interfin distance, concurrent flows for neighboring rows do not converge and are formed independently of one another.

In the general case, heat transfer in an individual fin is described by the equation

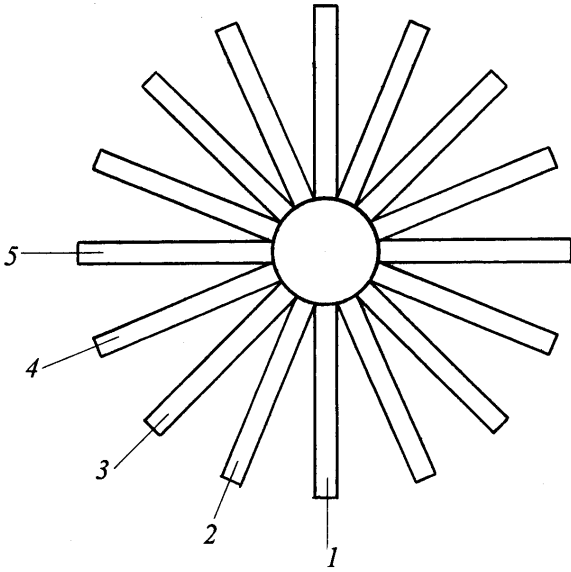


Fig. 1. Cylindrical surface with sectional finning.

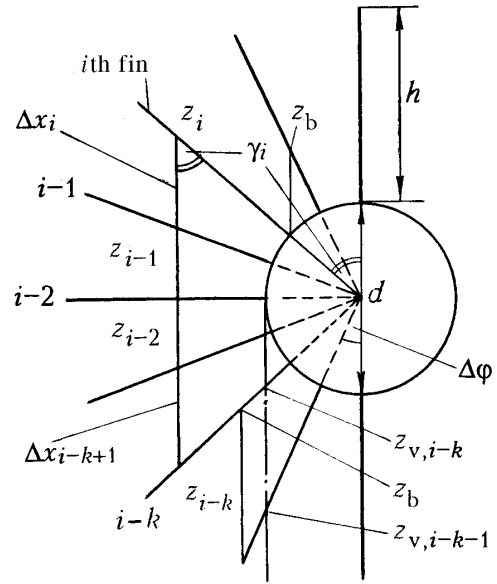


Fig. 2. Scheme of calculation of concurrent flows for the finned surface.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0. \quad (1)$$

Assuming that the heat loss from the end faces is insufficient and allowing for the symmetry of the system relative to the Oy axis, we write the boundary conditions for the individual fin:

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial T}{\partial x} \right|_{x=l} = 0, \quad \left. \frac{\partial T}{\partial y} \right|_{y=0} = 0, \quad \lambda_{\text{fin}} \left. \frac{\partial T}{\partial y} \right|_{y=\delta_{\text{fin}}/2} = q(x, z),$$

$$T(z=0) = T_0, \quad \left. \frac{\partial T}{\partial z} \right|_{z=h} = 0. \quad (2)$$

For thermally thin fins ($Bi = \alpha \delta_{\text{fin}} / \lambda_{\text{fin}} \ll 1$) with small cross-sectional dimensions the fin temperature changes slightly along the coordinates $0x$ and $0y$ and it can be averaged over the thickness and width of the fin; as a result, with the aid of dimensionless variables, Eq. (1) and boundary conditions (2) are reduced to the form

$$\frac{\lambda_{\text{fin}} \delta_{\text{fin}}}{2h^2} \frac{d^2 \theta}{dZ^2} = \tilde{q}(R, \gamma, \theta), \quad (3)$$

$$\theta(Z=0) = 1, \quad \left. \frac{\partial \theta}{\partial Z} \right|_{Z=1} = 0, \quad (4)$$

where

$$\theta = \frac{\tilde{T} - T_{\text{g}\infty}}{T_0 - T_{\text{g}\infty}}, \quad \tilde{q} = \frac{q(r, \gamma, \tilde{T})}{T_0 - T_{\text{g}\infty}}, \quad \tilde{T} = \frac{2}{\delta_{\text{fin}} l} \int_0^l dx \int_0^{\delta_{\text{fin}}/2} T(x, y, z) dy$$

is the fin temperature averaged over the thickness and width, $q(r, \gamma, \tilde{T})$ is the density of the heat flux removed from the fin surface, and $r(x, z)$ is the length of formation of the boundary layer on the fin surface.

Since the angle of inclination for the lower fin (fin 1 in Fig. 1) differs little from $\gamma \approx 180^\circ$ and heat transfer is determined only by natural convection, the density of the heat flux removed from its surface at an arbitrary temperature distribution is determined by the general functional dependence [3, 4]

$$q(z, \tilde{T}) = \alpha_n^*(h-z, \Delta T) \Delta T \left(\frac{1}{(h-z) \Delta T^{5/3}} \int_{h-z}^h \Delta T^{5/3} dz' \right)^{-1/4}, \quad (5)$$

where $\Delta T = \tilde{T} - T_{g\infty}$ is the temperature head on the fin surface. According to [5], the local heat-transfer coefficient on an isothermal surface is

$$\alpha_n^*(z, \Delta T) = c(\text{Pr}) \lambda_g (\text{Gr}_z \text{Pr})^{1/4} / z, \quad (6)$$

where the parameter $c(\text{Pr})$ depends on the Prandtl number and can be approximated by the expression

$$c(\text{Pr}) = \frac{3}{4} \left(\frac{\text{Pr}}{2.435 + 4.884 \text{Pr}^{1/2} + 4.953 \text{Pr}} \right)^{1/4}.$$

Relation (5) in dimensionless variables acquires the form

$$\tilde{q}(Z, \theta) = \alpha_n^*(1-Z, \theta) \theta \left(\frac{1}{(1-Z) \theta^{5/3}} \int_{1-Z}^1 \theta^{5/3} dZ' \right)^{-1/4}. \quad (7)$$

For fin 2 (Fig. 1), the main mechanism of heat transfer on its surface is natural convection of the external heat-transfer agent. Allowing for the small cross-sectional dimensions of fins and the small region of formation of the boundary layer, we assume the heat-transfer coefficient for the free-convective component of the heat flux to be virtually independent of a change in the temperature head:

$$\bar{\alpha}_n = \frac{1}{R-R_1} \int_{R_1}^R \alpha_n^*(\gamma, R', \theta) dR', \quad (8)$$

where α_n^* is determined by expression (6) in which the variable r is used instead of z (Fig. 3). The density of the heat flux removed from the surface of fin 2 is

$$\tilde{q}(Z, \theta) = \bar{\alpha}_n(Z, \theta) \theta. \quad (9)$$

The fins that lie above fin 2 (fins 3, 4, etc., as is shown in Figs. 1 and 2) are fully or partially in the concurrent flows from the fins lying below. On the surface of these fins, there is mixed convective flow and the density of the removed heat flux is determined from the formula

$$\tilde{q}(Z, \theta) = \alpha_m(Z, \theta) (\theta - \theta_g), \quad (10)$$

where $\theta_g = (T_g - T_{g\infty}) / (T_0 - T_{g\infty})$.

For the heat-transfer coefficient in mixed convection the relationship

$$\alpha_m = \alpha_n [1 + (\alpha_f / \alpha_n)^3]^{1/3} \quad (11)$$

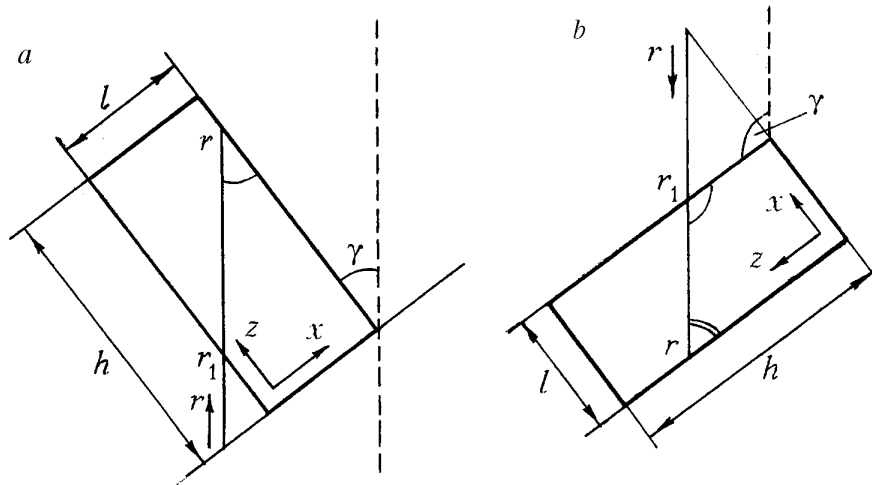


Fig. 3. Computation scheme for determining the length of formation of the boundary layer on an inclined fin: a) $0 < \gamma < 90^\circ$; b) $90^\circ < \gamma < 180^\circ$.

holds [6]; this relationship is true for both the local and surface-averaged quantities. The value of α_n in (11) is found from formulas (6) and (8), where T_g is used instead of $T_{g\infty}$. The forced-convective component of the local heat-transfer coefficient α_f is calculated from the dimensionless expression [7]

$$\alpha_f = 0.33 \lambda_g \text{Pr}^{0.33} \text{Re}_r^{1/2} / r, \quad (12)$$

and its mean value is found after integration of the local values along the length of the portion of boundary-layer formation r and is equal to $\bar{\alpha}_f = 2\alpha_f$.

A part of the surface of fin 3 is in the zone of concurrent flow from fin 2, and the free-convective mode is observed on the remaining part (see Figs. 1 and 2). A certain region of the surface of fin 4 near the base is in the concurrent flow from two heat sources (fins 2 and 3); then a part of the surface lies in the zone of the concurrent flow from fin 3, and natural-convective heat transfer occurs on the remaining part of the surface. The above regularity holds for fins lying in the lower part of the cylindrical base and whose angle of inclination is within $90^\circ \leq \gamma < 180^\circ$. It is assumed that fins can be considered as linear heat sources with an angle of inclination γ with the vertical, and the computation technique of [8] is used. The length of portions with mixed ($0 \leq z < z_b$) and free-convective ($z_b \leq z \leq h$) flows for the i th fin is found from the position of the crests of the closest lower ($i-1$)th fin relative to the i th fin. The coordinate of the separating boundary of the zones z_b is determined on the basis of the theorem of

sines, $z_b = \frac{(d/2 + h) \sin(180^\circ - \gamma - \Delta\phi)}{\sin \gamma} - \frac{d}{2}$. In calculating heat transfer for fins in the region $0 \leq z < z_b$, the density of

the removed heat flux is determined from formula (10) (in this case, use is made of the averaged values of the heat-transfer coefficients) and in the region of free-convective heat transfer $z_b \leq z \leq h$, $\tilde{q}(Z, \theta)$ is calculated from formula (9).

The fins lying in the upper part of the cylindrical base ($0 \leq \gamma < 90^\circ$) are entirely, in practice, under the conditions of mixed flow, except for a small portion of the near-root region where concurrent flow is absent. The length of this portion is $0 \leq z < z_b$ and the coordinate of separation of portions with different flow modes z_b is determined by the

position of the base of the nearest lower fin and is found using the theorem of sines, $z_b = \frac{d \sin(180^\circ - \gamma - \Delta\phi)}{2 \sin \gamma} - \frac{d}{2}$.

The technique of calculation of $\tilde{q}(Z, \theta)$ on the portions with different conditions of flow modes remains the same. We note that the model includes a certain idealization which does not correspond to the actual conditions of streamlining, viz., finite dimensions of fins are not taken into account in formation of concurrent flows and the effect of the boundary layer on the surface of the cylinder is neglected. By virtue of such assumptions, concurrent flow past the near-end region of a fin and flow in the boundary layer on the cylinder base are not calculated quite accurately.

To calculate $\tilde{q}(Z, \theta)$ from formula (10), in the mixed-flow region we must find the component $\bar{\alpha}_f$, the calculation of which necessitates determination of the velocity and the temperature in the concurrent flow formed from the lower-lying fins. According to the technique of [8], the system of fins which form the concurrent flow near the i th fin (reckoning from the lower fin with an angle of inclination of $\gamma = 180^\circ$, as shown in Fig. 2) can be treated as an effective linear heat source. The velocities of flow and the temperature on the axis line and also the boundary-layer thickness for the concurrent flow from the effective source are found from the expressions [8, 9]

$$U_{xi} = \left(\frac{2g\beta(Q_1 + Q_2 + \dots + Q_i)}{c_p I} \right)^{2/5} \left(\frac{x - x_k}{\mu \rho} \right)^{1/5} \left. \frac{\partial f_n}{\partial \eta} \right|_{\eta=0}, \quad (13)$$

$$T_{gi} - T_{g\infty} = \left(\frac{(Q_1 + Q_2 + \dots + Q_i)^4}{64g\beta\rho^2\mu^2 c_p I^4} \right)^{1/5} (x - x_k)^{-3/5}, \quad (14)$$

$$\delta_{gi} = \left(\frac{16\mu c_p I^2}{g\beta(Q_1 + Q_2 + \dots + Q_i)} \right)^{1/5} \eta_e (x - x_k)^{2/5}, \quad i = 1, 2, 3, \dots, \quad (15)$$

where f_n and η are the self-similar variables and I and η_e are the known calculated parameters for the concurrent flow from the linear heat source [9]. If the concurrent flow is formed by k fins (their number is determined by the intersection of the vertical dropped from the running point of the i th fin and the lower-lying fins, as shown in Fig. 2), then the distance x is reckoned from the $(i - k)$ th fin and x_k is the coordinate of displacement of the effective total heat source relative to the lower fin. In accordance with the approach of [8], x_k is found from the expression

$$x_k = \sum_{j=1}^k \Delta x_j - \left(1 + \frac{Q_k}{Q_1 + Q_2 + \dots + Q_{k-1}} \right)^{1/2} \left(\sum_{j=1}^k \Delta x_j - x_{k-1} \right), \quad j = 1, 2, \dots, k, \quad (16)$$

where k is the number of fins in the effective heat source. The distance along the vertical between the j th and the $(j - 1)$ th fins Δx_j is determined using the theorem of sines, $\Delta x_j = z \sin \Delta\phi / \sin(180^\circ - \gamma - \Delta\phi)$, where z is the running coordinate on the axis of the i th fin. The initial coordinate x_1 is assumed to be zero.

The power of an individual heat source is calculated through integration of the removed heat flux on the portion of formation of the boundary layer:

$$Q_i = 2 \int_{r_1}^r \alpha_i(r', \gamma) (\tilde{T}_i(r', \gamma) - T_{gi}) dr', \quad i = 1, 2, 3, \dots, \quad (17)$$

where $\alpha_i(r', \gamma)$ and $\tilde{T}_i(r', \gamma)$ are the local values of the heat-transfer coefficient and the temperature on the surface of the i th fin for the running variable r' . In this case, the quantity r depends on the running coordinate z and the angle of inclination of the fin γ and acquires the following values (as is shown in Fig. 3).

For the case where fins lie in the sector $\Delta\gamma < \gamma < 90^\circ$, where $\Delta\gamma = \arctan(l/h)$, we have $r_1 = 0$, $r = z/\cos \gamma$, $\tilde{T}(r', \gamma) = \tilde{T}(z')$, and $\alpha(r', \gamma) = \alpha(z'/\cos \gamma)$ on the portion of the fin $0 \leq z < l \cot \gamma$ and we find $r_1 = (z - l \cot \gamma)/\cos \gamma$, $r = z/\cos \gamma$, $\tilde{T}(r', \gamma) = \tilde{T}(z')$, and $\alpha(r', \gamma) = \alpha[(z' - z + l \cot \gamma)/\cos \gamma]$ in the region $l \cot \gamma \leq z \leq h$.

For the fins with an angle of $\gamma = 90^\circ$ we can write $r_1 = 0$, $r = l$, $\tilde{T}(r, \gamma) = T(z)$, and $\alpha(r', \gamma) = \alpha(z)$.

For the case where fins lie in the sector $90^\circ < \gamma < 180^\circ - \Delta\gamma$, on the portion of the fin $0 \leq z < h - l \cot \gamma$ we have

$$r_1 = z/\cos(180 - \gamma), \quad r = \frac{z + l \tan(180 - \gamma)}{\cos(180 - \gamma)}, \quad \tilde{T}(r', \gamma) = \tilde{T}(z'), \quad \alpha(r', \gamma) = \alpha \left[\frac{z + l \tan(180 - \gamma) - z'}{\cos(180 - \gamma)} \right],$$

and in the region $h - l \cot \gamma \leq z \leq h$ we find

$$r_1 = z, \quad r = h, \quad \tilde{T}(r', \gamma) = \tilde{T}(z'), \quad \alpha(r', \gamma) = \alpha \left[\frac{h - z'}{\cos(180 - \gamma)} \right].$$

Within the range of variation of the angle of inclination of fins $0 \leq \gamma \leq \Delta\gamma$ and $180^\circ - \Delta\gamma \leq \gamma \leq 180^\circ$, the variable r can be taken independent of γ , setting, respectively, $r_1 = 0, r = z$ and $r_1 = z, r = h$.

The entire upper fin lies in the concurrent flow arising in separation of the near-wall flow, which is formed on the cylindrical base due to the interaction of the boundary layer and the concurrent flows attached to its surface, from fins either totally or partially lying in the zone which is bounded by two vertical lines dropped from the points (for which $\gamma = 90^\circ$) on the base surface (see the dot-dash line in Fig. 2). The value of the coordinate of the point of intersection of the vertical and the i th fin is $z_{v,i} = d/[2 \sin(180^\circ - \gamma_i)]$. In order to calculate the velocity and the temperature in the concurrent flow above the cylindrical base, according to [8] we must find the value of the power of the effective heat source, which is calculated through summation of the powers of individual heat sources:

$$Q_s = Q_c + 2 \sum_{i=1}^n \bar{Q}_i, \quad (18)$$

where i is the number of a fin and n is the number of fins lying in half the indicated zone. The power of individual sources in the zone under consideration is calculated through integration of the local heat fluxes removed from the corresponding portions of the fin surface:

$$\bar{Q}_i = 2 \int_0^{z_{v,i}} dz \int_{r_1}^r \alpha_i(r', \gamma) (\tilde{T}_i(r', \gamma) - T_{gi}) dr', \quad i = 1, 2, \dots, n, \quad (19)$$

(for fins lying totally in the assigned zone we have $z_{v,i} = h$ and for fins partially entering the assigned region the values of $z_{v,i}$ are found above).

The power of the cylindrical base Q_c is

$$Q_c = \pi d \bar{\alpha}_n (T_0 - T_{g\infty}), \quad (20)$$

where $\bar{\alpha}_n$ is the averaged coefficient of heat transfer on the surface of the cylindrical base, which will be determined below.

The coordinate of the position of the effective heat source relative to the base of the upper fin x_s can be found from the condition of equality of the thicknesses of the boundary layer formed on the cylindrical base at the point of separation and the boundary layer of the concurrent flow

$$\delta_{gs}(x_s) = \delta_{g,\text{layer}}, \quad (21)$$

where the thickness of the boundary layer of the concurrent flow δ_{gs} is determined from formula (15) where the value of the power of the effective source is determined by expression (18) and x_k is taken to be zero. The thickness of the boundary layer on the cylinder surface at the point of separation $\delta_{g,\text{layer}}$ is determined from the known results of the calculation of heat transfer on the cylinder surface in free convection [5] and the relation between the boundary-layer thickness and the local coefficient of heat transfer $\delta_{g,\text{layer}}(x) = \lambda_g / \alpha_n(x)$. For a horizontal cylinder, the local coefficient of heat transfer is determined by the dependence [5]

$$\alpha_n(x) = \lambda_g f(x/d, \text{Pr}) \text{Gr}_d^{1/4} / d, \quad (22)$$

which upon averaging over the surface acquires the form

$$\bar{\alpha}_n = c(\text{Pr}) \lambda_g \text{Gr}_d^{1/4} / d. \quad (23)$$

The coefficient $c(\text{Pr})$ in (23) depends on the Prandtl number and can be found by both computation and experiment [5].

With account for (22), the thickness of the boundary layer on the cylinder surface is

$$\delta_{\text{g.layer}}(x) = d \text{Gr}_d^{-1/4} / f(x/d, \text{Pr}), \quad (24)$$

where $x = d(\pi - \gamma)/2$ is the length of formation of the boundary layer on the cylinder surface (the angle γ is expressed in radians) and the function $f(x/d, \text{Pr})$ is determined in solving the equation of transfer for the boundary layer on the cylinder surface. As an example, we give the expression for the function $f(x/d, \text{Pr})$ obtained in [10] for the particular case $T = \text{const}$ and $\text{Pr} = 0.7$: $f(x/d, \text{Pr}) = 0.37023 - 0.06436(x/d)^2 - 0.00144(x/d)^4$. From equality (21), with relations (15) and (24) at the point of separation of the boundary layer $x = d\pi/2$ in mind, we find the coordinate of the position of the effective heat source:

$$x_s = d \left(\frac{16\mu c_p T^2}{g\beta Q_s} \right)^{-2} \eta_e^{-5/2} \text{Gr}_d^{-1/4} / f(\pi, \text{Pr}). \quad (25)$$

We can note a certain difference between the thicknesses of the boundary layer for the cylinder (used in calculations) and of the near-wall flow developing on the cylinder surface in interaction between the boundary layer and the attached concurrent flows from the fins lying under the cylinder base. However, determination of this quantity is a separate problem and is beyond the scope of the present study.

Thus, calculation for the upper fin is carried out in the following order. Having determined the powers of individual heat sources from formulas (19) and (20), the power of the effective heat source from formula (18), and its position from relation (25), we can calculate the velocity and temperature of the concurrent flow from formulas (13) and (14), where $x_k = x_s$. Then we determine the local values of the coefficients of heat transfer for free and forced convection from formulas (6) and (12) and the total value of this coefficient from relation (11) and find the local removed heat flux according to (10).

In numerical calculation of the equation of transfer (3) with boundary conditions (4) for each fin we used the Runge–Kutta scheme and calculated the integrals by the method of trapezoids [11]. Since in relation (5) the temperature head $\Delta T = \tilde{T} - T_{g\infty}$ over the height of fins is a priori unknown, we organized the iteration procedure of calculations. In calculations, we set $\Delta T = T_0 - T_{g\infty}$ in the first iteration with further verification of ΔT in the following iterations. During the calculation, we determined the power of the heat source represented by an individual fin and also the velocities and temperatures of the heat-transfer agent in concurrent flows and the coordinates of the position of the effective heat sources. Calculation was made successively from fin to fin starting with the lower one. The boundary of separation portions with different conditions of heat transfer was determined for each fin and corresponding relations were used to calculate the removed heat fluxes on each portion. We found the local and integral thermal characteristics for each fin and the total removed heat flux for the entire system.

Some results of numerical calculations for a heated cylinder with air-cooled steel sectional finning ($\lambda_{\text{fin}} = 40 \text{ W}/(\text{m}^2 \cdot \text{K})$) are presented in Figs. 4–6. It was assumed that with the outside diameter of the cylindrical base $d = 13 \text{ mm}$ fins have dimensions $h \times l \times \delta_{\text{fin}} = 22 \times 3 \times 0.5 \text{ mm}$ and the interfin space is $s = 3 \text{ mm}$. Figure 4 gives the dependence of the Nusselt number Nu_d averaged over the entire surface on the Rayleigh number Ra_d , which is obtained from calculations and experiments [12]. Comparison of the calculated and experimental data points to a certain understatement of the calculated values with an error of 14–23%.

Figure 5 shows the distributions of the relative coefficients of heat transfer $\alpha/\bar{\alpha}_n^*$ over the height of fins with an angle of inclination of $\gamma = 0, 45, 90, 135, \text{ and } 180^\circ$, which are obtained by calculations at $T_0 = 100^\circ\text{C}$ and $T_{g\infty} = 20^\circ\text{C}$. The mean value of the heat-transfer coefficient for an isothermal fin $\bar{\alpha}_n^*$ obtained through averaging of its local values (6) over the fin height at $\Delta T = T_0 - T_{g\infty}$ is $\bar{\alpha}_n^* = \frac{3}{4} c(\text{Pr}) \lambda_g (\text{Gr}_h \text{Pr})^{1/4} / h$. The same figure also gives the rela-

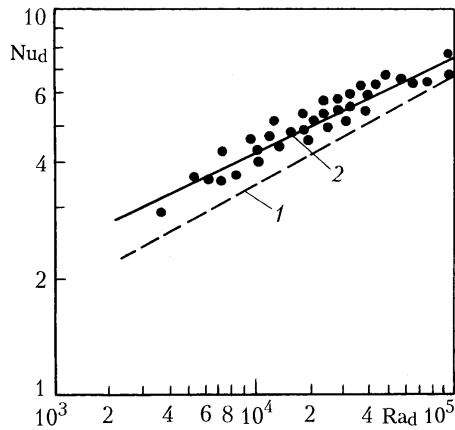


Fig. 4. Dependence of the averaged Nusselt number Nu_d on the Rayleigh number Ra_d : 1) calculation; 2) experiment [12].

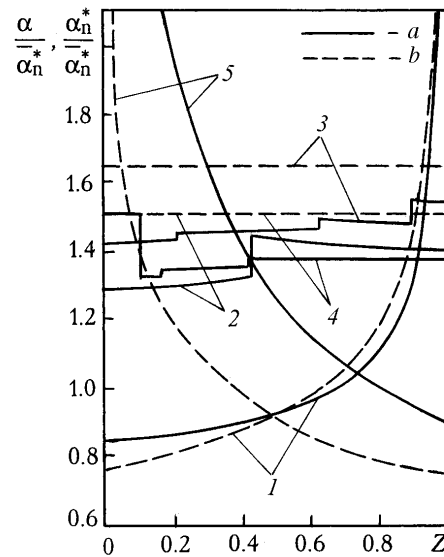


Fig. 5. Distributions of the coefficients of heat transfer over the height of fins [a) calculation of α/α_n^* ; b) calculation of α_n^*/α_n^*]: 1) fin with an angle of inclination of $\gamma = 180^\circ$; 2) 135; 3) 90; 4) 45; 5) 0.

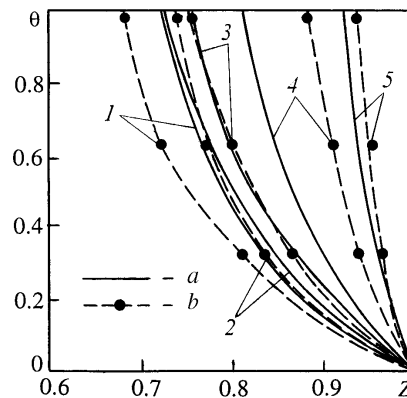


Fig. 6. Temperature distributions θ over the height of fins (a, calculation; b, experiment [12]): 1) fin with an angle of inclination of $\gamma = 180^\circ$; 2) 135; 3) 90; 4) 45; 5) 0.

tive coefficients of heat transfer α_n^*/α_n^* for individual isothermal fins with a surface temperature of $\tilde{T} = T_0$, which have the same position in the surrounding medium in free convection (without regard for concurrent flows from lower-lying sources). As follows from Fig. 5, the distributions of the heat-transfer coefficients over the height of fins (except for the upper and lower ones) have a stepwise character. This is caused by the different number of lower-lying heat sources and stepwise changes of the velocity and the temperature in concurrent flows past different portions over the fin height. The distribution of the heat-transfer coefficients over the height of the lower and upper fins has a monotonic character. Comparison of the distributions obtained for the finned system under consideration with similar distributions over the height of individual isothermal fins having the same angle of inclination indicates that they have lower values for sectional finning. This is due to the substantial heating of the heat-transfer agent in concurrent flows and, consequently, the decrease in the free-convective component of the heat-transfer coefficient, which is predominant for these fins. The exception is the upper fin, for which the free-convective component is larger. At the same time, for the upper fin the temperature head on the surface is insignificant due to the substantial heating of the heat-transfer agent in the concurrent flow, and the local values of the heat fluxes removed from its surface are low.

Figure 6 gives the calculated and experimental [12] temperature distributions over the height of fins arranged at different angles of inclination. An analysis of the curves obtained indicates a substantial decrease in the temperature head for the fins lying in the lower part of the cylindrical base $90^\circ \leq \gamma \leq 180^\circ$. We have a somewhat smaller decrease in the head for the fins with angles of inclination of $\gamma = 90^\circ$ and $\gamma = 135^\circ$, and for the upper fin ($\gamma = 0$) the change in the temperatures over the height is insignificant. Comparison of the calculated distributions obtained and the experimental data points to their good agreement.

NOTATION

x, y, z , running coordinates; T , temperature; q , density of the removed heat flux; d , diameter of the cylindrical base; h and l , height and width of fins; s , interfin space; U , velocity of flow; g , gravitational acceleration; Q , total heat flux or power of the heat source; γ , angle of inclination of the axial line of the fin relative to the vertical; $\Delta\phi$, angular distance between the fins; δ , thickness; ρ , density of the external heat-transfer agent; λ , thermal conductivity; α , local coefficient of heat transfer; μ , kinematic viscosity; ν , dynamic viscosity; β , coefficient of volumetric expansion; c_p , specific heat capacity; f and η , self-similar variables for concurrent flow; $Z = z/h$, $R = r/h$; $\theta = (T - T_{g\infty}) / (T_0 - T_{g\infty})$, dimensionless temperature; Pr, Prandtl number; $Bi = \alpha\delta/\lambda$, Biot number; $Gr_x = g\beta(T - T_g)x^3/\nu^2$, Grashof number; $Re_x = Ux/\nu$, Reynolds number; $Ra_x = Gr_x Pr$, Rayleigh number; I , calculated parameter of concurrent flow. Sub- and superscripts: *, isothermal surface; fin, fin; n, natural convection; f, forced convection; m, mixed convection; 0, fin base; ∞ , surrounding medium; g, external heat-transfer agent; b, boundary separating portions with different conditions of flow; s, effective (total) heat source; e, edge of the boundary layer; c, cylindrical surface; layer, boundary layer; d, diameter; v, points of intersection of fins and the vertical; ', running value of the variable in the integral.

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